

CLASSICAL AND QUANTAL
ASPECTS OF
ROTATING ATOMIC NUCLEI

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by

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Civ.ing., Lnd.**

Varje generellt påstående är som en check utställd på en bank.
Dess värde beror på vilken täckning den har.

E. Pound

This thesis is comprised by this introduction and summary, and the following articles

- I. P. Arve and H. Reinhardt, Nuclear Rotation in the Quantized TDHF Theory, Physics Letters 105B (1981) 249
- II. P. Arve, Y.S. Chen and G.A. Leander, Microscopic Calculation of Nuclear Wobbling Rotation, Physica Scripta T5 (1983) 157
- III. P. Arve, Nuclear Field Theory Treatment of $K \neq 0$ Rotational Bands, submitted to Nuclear Physics
- IV. P. Arve, A Collective Contribution to $M1$ Transitions in the Cranking model, to be submitted to Physics Letters

1. Introduction

The rapid growth of experimental knowledge of the properties of rotating nuclei is impressive. It serves as an important test of our understanding of nuclear structure and promises us new insights. This dissertation is mainly a discussion of some methods for calculating energies, moments and transition rates of nuclear high-spin states.

Studies of nuclear structure aim to understand the more or less coherent motion of the particles forming the nucleus. Since the 1930s the nucleus has been viewed as an aggregate of protons and neutrons, which are commonly called nucleons. The different kinds of nuclei are since then identified with the total charge Z and the total number of nucleons A . During the 1940s and 1950s it was established that the strong interaction felt by the nucleons are intermediated by mesons. Unlike the number of nucleons, the number of mesons is not fixed. Actually, the mesons present in the nucleus are often assumed to be virtual, i.e. they exist only as quantum fluctuations. In the 1960s and the 1970s the picture of strongly interacting particles as composed of quarks and gluons became evident.

The nucleus can in principle be described in different terms, for example as quarks and gluons, as nucleons and mesons, as nucleons interacting via two-body and many-body forces, or as nucleons moving in a common potential. Which is the most appropriate way depends on what aspect of the nucleus is studied. It is obvious that the high-spin phenomena under current investigation primarily are connected with more or less collective motion of nucleons.

Sometimes it is preferable to further simplify and consider the whole or most of the nucleus as a lump of continuous matter like a rigid body or a liquid drop. The parametrization of the

binding energy in terms of volume, surface and Coulomb contributions to the energy of a liquid drop [We35,BB36] was important for the understanding of fission [BW39]. It has also been used to estimate the stability of rapidly rotating nuclei [CP74].

The large quadrupole moments of some heavy nuclei led J. Rainwater [Ra50] and A. Bohr [Bo51] to assume that the nucleons experience a quadrupole deformed potential. In the following discussions of the vibrations and rotations of the potential [Bo52,BM53] the dynamics was identified with that of a liquid drop or a rigid body, which enabled a proper quantal treatment. By use of the cranking method proposed by D.R. Inglis [In54] it was demonstrated by A. Bohr and B. Mottelson [BM55] that the response of the nucleons to rotation is such that the moment of inertia should be given by that of a rigid body with the same mass distribution. This holds if the potential is local, the nucleus is heavy and the energy is minimized. However, pairing correlations which are not determined by the average potential are responsible for the fact that heavy nuclei can have moments of inertia appreciably smaller than the rigid body value [BM58].

It is clear from what is mentioned above that it is useful to resort to classical pictures even in the discussion of phenomena where quantal aspects are important. This has been especially significant in the understanding of collective motion. Partly this is due to the simplicity of the collective motion. For instance, for harmonic vibrations or slow rotations the quantization leads to such results that we can easily translate between quantal and classical descriptions.

Concerning rotations a further observation [KK63] is interesting. The very small energy differences between the quantal levels justifies the construction of an approximate eigenstate as a superposition of true eigenstates. Especially, if the rotation is (almost) adiabatic, i.e. the moment of inertia is (almost) infinite, then a state can be built which

has a well defined direction in space. This state can be identified with the classical rotor pointing in the same direction in space. It embodies correlations present in all of the true eigenstates. Its different electromagnetic moments are connected with the electromagnetic transition matrix elements within the rotational band. Such a state is likely to be low in energy, in the Hartree (-Fock) approximation, as only those correlations are counted that give rise to nonvanishing expectation values of single-particle operators. A model for this state was developed by S.G. Nilsson [Ni55].

The path integral formulation of quantum mechanics as given by R.P. Feynman [Fe48] explains the connection between quantal and classical descriptions in an appealing way. The amplitude for a particle to move from point a at time t_1 and hit point b at time t_2 is given as a sum or rather an infinite dimensional integral over all possible continuous paths $x(t)$, not only the solution to the classical equations of motion. Each path contribute

$$\exp\left(i\frac{S}{\hbar}\right)$$

where S is the action for that path.

By applying the stationary phase approximation to the integral, only contributions from the classical solution and neighbouring paths are retained. Thus the classical approximation is good when a small variation in the path gives rise to a variation in the action which is large compared with $\hbar$.

A path integral formulation of the partition function of an interacting many-body system was given by R.L. Stratanovich [St57]. J. Hubbard [Hu59] used this formulation to describe the dynamics of a many-body system by taking the temperature to be imaginary. In this description the two-body interaction is replaced by a quantal (nuclear) field which interacts with the nucleons. That the field is quantal means that it is an integration variable. The (semi-)classical motion of the field

is given by the time dependent Hartree equation [Re79]. Within this formalism, a Bohr-Sommerfeld condition for the quantization of periodic classical motion was derived by H. Reinhardt [Re80] and S. Levit, J. Negele and Z. Paltiel [LN80].

2. Summary

Already in the early 1960s it was shown by D.J. Thouless and J.G. Valatin [TV62] that the cranking model [In54] could be derived from the TDH(-F) equations. When they transformed the equations into an uniformly rotating coordinate system and looked for solutions with a time independent density matrix the cranking model appeared. Thus a (self-consistent) solution of the cranking equations define a TDH(-F) motion such that the nuclear field and thereby the nucleons rotate around an axis fixed in space.

The classical nature of the TDH(-F) equations is also present in the cranking model. For instance, the length of the angular momentum vector can be continuously increased if not supplementary conditions are given. Nevertheless, it is a powerful tool for the prediction and explanation of major changes in the nuclear field with increasing angular momentum. It might fail in the detailed description of processes in which the quantal structure of the single-particle orbit play a dominant role [Ha76].

As discussed in paper I the TDH(-F) orbit is periodic, i.e. the density matrix is periodic. The quantization condition was applied in a straightforward manner. It resulted in a condition for the length of the angular momentum I . The precise rule depends on the symmetries of the cranking wave function. If the wave function is an eigenstate to the rotation around the "cranking axis" by 180° , i.e. signature is a good quantum number, the frequency of the orbit is twice the cranking frequency and the corresponding action equals $2I$. More

precisely, the quantization amounts to $I=a \bmod 2$, where a is the signature. Finally, assuming good signature and an approximate symmetry axis perpendicular to the cranking axis, it was argued that the cranking model first of all corresponds to $K=0$ or $K=1/2$.

A very different situation is encountered in high- K rotational bands. It corresponds to the motion of a symmetric top whose symmetry axis precesses around the total angular momentum. In paper II, two different methods for the calculation of high- K bands were discussed and used to estimate the moments of inertia for several measured and not yet measured bands. One of the methods is based on the coupling of several particles in a high- j shell to a symmetric rotor with a fixed angular momentum along the symmetry axis. This model is schematic in the treatment of the effect of the particles constituting the core, but completely quantal. The diagonalization of the Hamiltonian was performed in a basis which was rather limited compared with the complete space. The other method is closer related to the classical motion of a rotor with large angular momentum along a symmetry axis and a comparatively small perpendicular angular momentum component. All the particles in the system are explicitly taken into account, and treated equally. It is assumed that the amplitude of the motion of the field is small, so that it can be treated within the vibrational formalism denoted RPA. The calculation differed from previous calculations [AK81, AB81] in the choice of residual interaction. It was defined so that the entire potential was moving together with the nucleons and not only the major part of it. The energy spacing between the band head and the next lowest member of the rotational band was calculated for high- K states in ^{175}Hf , ^{176}Hf , ^{179}W and ^{158}Yb .

Paper III has two purposes. Firstly, to confirm the interpretation of some solutions to the small amplitude vibrations as describing rotational motion. Secondly, to show that the coupling between the quantized vibrations and the nucleons induce anharmonicities typical for rotational motion,

thus constituting a purely microscopic and quantal scheme for the calculation of rotational motion. The rules for a consistent calculation of the effects from particle-vibration coupling were sketched by B. Mottelson [Mo68] and later studied in detail by many authors and often referred to as Nuclear Field Theory. This method was applied to an exactly solvable model formulated by J.P. Elliott [El58]. The spectrum is very similar to that of a rotor, hence this model can be a fruitful testing ground for approximate calculations of rotational motion.

As a specific example a $K=4$ state in ^{24}Mg was chosen. The energies of the states in the $K=4$ band and adjacent bands were reproduced. It was shown that the perturbation series must be regrouped into an expansion in L , a parameter related to the number of degrees of freedom participating. It was also shown that the coupling between one particle and the rotational motion of the other particles can be calculated microscopically.

In paper IV a contribution from collective rotation to some transitions changing the angular momentum by one unit was derived. E. Marshalek [Ma77] has shown that a complete RPA treatment of the excitations of a cranking state will always give one mode connected with the reorientation of the nucleus. The polarisation contributions from this mode to one quasiparticle transitions was studied. Simple expressions for the M1 and E2 transitions appeared. Of special interest is the simple understanding of the structure of E2 transitions earlier seen in particle triaxial-rotor calculations [HM83]. A similar structure has been seen in a recent experiment [HG82].

Finally, I wish to thank those that helped and inspired me during this work, especially Professor Ben Mottelson.

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